

**Some Formulae**

$$\begin{aligned}
\text{Poisson}(\alpha) : & \quad p_X(k) = \frac{\alpha^k e^{-\alpha}}{k!}, \quad k = 0, 1, 2, \dots, & \quad E[X] = \alpha, \quad \text{var}[X] = \alpha \\
\text{Geometric}(p) : & \quad p_X(k) = (1-p)^{k-1} p, \quad k = 1, 2, \dots, & \quad E[X] = \frac{1}{p}, \quad \text{var}[X] = \frac{1-p}{p^2}. \\
\text{Binomial}(n, p) : & \quad p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad 0 \leq k \leq n, & \quad E[X] = np, \quad \text{var}[X] = np(1-p). \\
\text{Exponential}(\lambda) : & \quad f_X(x) = \lambda e^{-\lambda x} \text{ and } F_X(x) = 1 - e^{-\lambda x}, & \quad E[X] = \frac{1}{\lambda}, \quad \text{var}[X] = \frac{1}{\lambda^2}.
\end{aligned}$$

Moment generating function (mgf):  $\phi_X(s) = E[e^{sX}]$ .

Binomial coefficient:

$$\binom{n}{k} = \binom{n}{n-k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)\cdots(n-k+1)}{k!}$$

**Bayes' Rule:** Given a partition  $B_1, B_2, \dots, B_n$  of the sample space and an event  $A$ ,

$$P[B_j|A] = \frac{P[A|B_j] \cdot P[B_j]}{\sum_{k=1}^n P[A|B_k] \cdot P[B_k]}$$

If  $X \sim \mathcal{N}(m, \sigma^2)$  then  $f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2\right)$ .

If  $\mathbf{X} \sim \mathcal{N}(\mathbf{m}, \mathbf{K})$  is a Gaussian random vector with mean vector  $\mathbf{m}$  and covariance matrix  $\mathbf{K}$ , then

$$f_{\mathbf{X}}(\mathbf{x}) = \frac{\exp\left\{-\frac{1}{2}(\mathbf{x} - \mathbf{m})^T \mathbf{K}^{-1}(\mathbf{x} - \mathbf{m})\right\}}{(2\pi)^{n/2} |\det \mathbf{K}|^{1/2}}$$

2-by-2 Matrix Determinant and Inverse

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \det \mathbf{A} = ad - bc, \quad \mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Trigonometric Formulae:

$$\begin{aligned}
\cos(s+t) &= \cos s \cos t - \sin s \sin t, \\
\sin(s+t) &= \sin s \cos t + \cos s \sin t.
\end{aligned}$$

$\Phi$ -table and  $Q$ -table over the page

# 1 Basics

Conditional Probability

$$P[A|B] = \frac{P[A \cap B]}{P[B]}$$

Bayes' Rule

$$P[B|A] = \frac{P[A|B]P[B]}{P[A]}$$

Scaling and Shifting

$$Y = aX + b$$

$$f_Y(y) = \frac{dF_Y(y)}{dy} = \frac{dF_X(\frac{y-b}{a})}{d(\frac{y-b}{a})} = \frac{1}{a} f_X(\frac{y-b}{a})$$

$$f_Y(y) = \frac{1}{|a|} f_X(\frac{y-b}{a})$$

# 2 Continuous RV's

# 3 Transformations

General Transformation

$$f_Y(y) = |\det \frac{\delta x}{\delta y}| f_X(g^{-1}(y))$$

Affine Transformation

$$W = AX + b$$

$$f_W(w) = \frac{1}{|\det(A)|} f_X(A^{-1}(w - b))$$

Gaussian Transformations

$$W = AX + b$$

$$\mu_W = A\mu_X + b$$

$$C_W = AC_X A^T$$

Polar coordinate transformation

$$X_1 = R \cos(\theta)$$

$$X_2 = R \sin(\theta)$$

$$\frac{\delta(x_1, x_2)}{\delta(r, \theta)} = \begin{bmatrix} \cos(\theta) & -r \sin(\theta) \\ \sin(\theta) & r \cos(\theta) \end{bmatrix}$$

$$Det = r$$

$$f_{[R, \Theta]}(r, \theta) = r f_{[X_1, X_2]}(r \cos(\theta), r \sin(\theta))$$

Box-Muller Transformation

$$X_1 = \sqrt{V} \cos(\theta)$$

$$X_2 = \sqrt{V} \sin(\theta)$$

$$\frac{\delta(x_1, x_2)}{\delta(r, \theta)} = \begin{bmatrix} \frac{1}{2} \frac{\cos(\theta)}{\sqrt{v}} & -\sqrt{v} \sin(\theta) \\ \frac{1}{2} \frac{\sin(\theta)}{\sqrt{v}} & \sqrt{v} \cos(\theta) \end{bmatrix}$$

$$Det = \frac{1}{2}$$

$$f_{[V, \Theta]}(v, \theta) = \frac{1}{2} f_{[X_1, X_2]}(\sqrt{v} \cos(\theta), \sqrt{v} \sin(\theta))$$

# 4 Mean and Covariance

Correlation Matrix

$$R_X = E[XX^T]$$

Covariance

$$Cov[X, Y] = E[(X - \mu_X)(Y - \mu_Y)]$$

Covariance Matrix

$$\begin{aligned} C_X &= E[(X - \mu_x)(X - \mu_x)^T] \\ &= E[XX^T] - \mu_x \mu_x^T \\ &= R_X - \mu_x \mu_x^T \\ &= \begin{bmatrix} Var[X] & Cov[X, Y] \\ Cov[X, Y] & Var[Y] \end{bmatrix} \end{aligned}$$

Correlation between two RV's

$$E[XY] = E[X]E[Y] \text{ For independent RV's}$$

Correlation Coefficient

$$\begin{aligned} \rho_{X, Y} &= \frac{Cov[X, Y]}{\sqrt{Var[X]Var[Y]}} \\ &= \frac{Cov[X, Y]}{\sigma_X \sigma_Y} \end{aligned}$$

Cross-Covariance Matrix

$$C_{XW} = E[(X - \mu_X)(W - \mu_W)^T]$$

Cross-Correlation Matrix

$$E[XW^T]$$

Note that the auto-covariance is same as the covariance matrix, and the auto-correlation matrix is the same as the correlation matrix

## 5 Conditional RV

Conditional Joint CDF

$$F_{(X,Y)|B}(x, y) = P[X \leq x \text{ and } Y \leq |B]$$

Conditional Joint PDF

$$f_{(X,Y)|B}(x, y) = \begin{cases} \frac{f_{X,Y}(x,y)}{P[B]} & (x, y) \in B \\ 0 & \text{otherwise} \end{cases}$$

Conditional Joint PMF

$$p_{(X,Y)|B}(x, y) = \begin{cases} \frac{p_{X,Y}(x,y)}{P[B]} & (x, y) \in B \\ 0 & \text{otherwise} \end{cases}$$

Conditional Expected Value Function

$$E[X|Y = y] = \sum_{x \in S_X} x p_{X|Y}(x|y)$$

$$E[X|Y = y] = \int_{-\infty}^{\infty} x f_{X|Y}(x|y) dx$$

$$E[E[X|Y]] = E[X]$$

Handy Fact

$$E[E[g(X)|Y]] = E[g(X)] \text{ if } g(\cdot) \text{ is a function}$$

## 6 Sum of Random Variables

Central Limit Theorem

$X_j$  i.i.d

$$\sigma_X^2 = Var[X]$$

$$Z_n = \frac{(X_1 + X_2 + \dots + X_n) - nE[X]}{\sqrt{n}}$$

$$E[Z_n] = 0$$

$$Var[Z_n] = \sigma_X^2$$

$$\lim_{n \rightarrow \infty} F_{Z_n}(z) = F_G(z)$$

## 7 Random Processes

Auto-Correlation Function

$$R(t_1, t_2) = E[X(t_1)X(t_2)]$$

Auto-Covariance Function

$$K(t_1, t_2) = Cov[X(t_1), X(t_2)]$$

$$= R(t_1, t_2) - \mu(t_1)\mu(t_2)$$

White Noise Process

$$\mu = 0$$

$$R(\tau) = K\delta(\tau) \text{ where } k \text{ is a constant}$$

For discrete time, and iid zero-mean random sequence is white noise

$$R_X(\tau) = 0, \tau \neq 0$$

$$\sigma_X^2 = R_X(0) \text{ (Process variance)}$$

In the CT time case, any two  $X(t)$  and  $X(t+\tau)$  are uncorrelated

A stationary random process has

- Constant mean and constant variance
- Auto correlation function that can be written as a function of just one variable

$R(\tau) = E[X(t + \tau)X(t)]$  is only dependent on  $\tau$   
For example, i.i.d random sequence, white noise process

A wide-sense stationary process has

- Constant mean
- Auto correlation function that can be written as a function of just the time difference  $t_1 - t_2$

$$R(\tau) = E[X(t + \tau)X(t)]$$

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$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$
0.00	0.5000	0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.97725	2.50	0.99379
0.01	0.5040	0.51	0.6950	1.01	0.8438	1.51	0.9345	2.01	0.97778	2.51	0.99396
0.02	0.5080	0.52	0.6985	1.02	0.8461	1.52	0.9357	2.02	0.97831	2.52	0.99413
0.03	0.5120	0.53	0.7019	1.03	0.8485	1.53	0.9370	2.03	0.97882	2.53	0.99430
0.04	0.5160	0.54	0.7054	1.04	0.8508	1.54	0.9382	2.04	0.97932	2.54	0.99446
0.05	0.5199	0.55	0.7088	1.05	0.8531	1.55	0.9394	2.05	0.97982	2.55	0.99461
0.06	0.5239	0.56	0.7123	1.06	0.8554	1.56	0.9406	2.06	0.98030	2.56	0.99477
0.07	0.5279	0.57	0.7157	1.07	0.8577	1.57	0.9418	2.07	0.98077	2.57	0.99492
0.08	0.5319	0.58	0.7190	1.08	0.8599	1.58	0.9429	2.08	0.98124	2.58	0.99506
0.09	0.5359	0.59	0.7224	1.09	0.8621	1.59	0.9441	2.09	0.98169	2.59	0.99520
0.10	0.5398	0.60	0.7257	1.10	0.8643	1.60	0.9452	2.10	0.98214	2.60	0.99534
0.11	0.5438	0.61	0.7291	1.11	0.8665	1.61	0.9463	2.11	0.98257	2.61	0.99547
0.12	0.5478	0.62	0.7324	1.12	0.8686	1.62	0.9474	2.12	0.98300	2.62	0.99560
0.13	0.5517	0.63	0.7357	1.13	0.8708	1.63	0.9484	2.13	0.98341	2.63	0.99573
0.14	0.5557	0.64	0.7389	1.14	0.8729	1.64	0.9495	2.14	0.98382	2.64	0.99585
0.15	0.5596	0.65	0.7422	1.15	0.8749	1.65	0.9505	2.15	0.98422	2.65	0.99598
0.16	0.5636	0.66	0.7454	1.16	0.8770	1.66	0.9515	2.16	0.98461	2.66	0.99609
0.17	0.5675	0.67	0.7486	1.17	0.8790	1.67	0.9525	2.17	0.98500	2.67	0.99621
0.18	0.5714	0.68	0.7517	1.18	0.8810	1.68	0.9535	2.18	0.98537	2.68	0.99632
0.19	0.5753	0.69	0.7549	1.19	0.8830	1.69	0.9545	2.19	0.98574	2.69	0.99643
0.20	0.5793	0.70	0.7580	1.20	0.8849	1.70	0.9554	2.20	0.98610	2.70	0.99653
0.21	0.5832	0.71	0.7611	1.21	0.8869	1.71	0.9564	2.21	0.98645	2.71	0.99664
0.22	0.5871	0.72	0.7642	1.22	0.8888	1.72	0.9573	2.22	0.98679	2.72	0.99674
0.23	0.5910	0.73	0.7673	1.23	0.8907	1.73	0.9582	2.23	0.98713	2.73	0.99683
0.24	0.5948	0.74	0.7704	1.24	0.8925	1.74	0.9591	2.24	0.98745	2.74	0.99693
0.25	0.5987	0.75	0.7734	1.25	0.8944	1.75	0.9599	2.25	0.98778	2.75	0.99702
0.26	0.6026	0.76	0.7764	1.26	0.8962	1.76	0.9608	2.26	0.98809	2.76	0.99711
0.27	0.6064	0.77	0.7794	1.27	0.8980	1.77	0.9616	2.27	0.98840	2.77	0.99720
0.28	0.6103	0.78	0.7823	1.28	0.8997	1.78	0.9625	2.28	0.98870	2.78	0.99728
0.29	0.6141	0.79	0.7852	1.29	0.9015	1.79	0.9633	2.29	0.98899	2.79	0.99736
0.30	0.6179	0.80	0.7881	1.30	0.9032	1.80	0.9641	2.30	0.98928	2.80	0.99744
0.31	0.6217	0.81	0.7910	1.31	0.9049	1.81	0.9649	2.31	0.98956	2.81	0.99752
0.32	0.6255	0.82	0.7939	1.32	0.9066	1.82	0.9656	2.32	0.98983	2.82	0.99760
0.33	0.6293	0.83	0.7967	1.33	0.9082	1.83	0.9664	2.33	0.99010	2.83	0.99767
0.34	0.6331	0.84	0.7995	1.34	0.9099	1.84	0.9671	2.34	0.99036	2.84	0.99774
0.35	0.6368	0.85	0.8023	1.35	0.9115	1.85	0.9678	2.35	0.99061	2.85	0.99781
0.36	0.6406	0.86	0.8051	1.36	0.9131	1.86	0.9686	2.36	0.99086	2.86	0.99788
0.37	0.6443	0.87	0.8078	1.37	0.9147	1.87	0.9693	2.37	0.99111	2.87	0.99795
0.38	0.6480	0.88	0.8106	1.38	0.9162	1.88	0.9699	2.38	0.99134	2.88	0.99801
0.39	0.6517	0.89	0.8133	1.39	0.9177	1.89	0.9706	2.39	0.99158	2.89	0.99807
0.40	0.6554	0.90	0.8159	1.40	0.9192	1.90	0.9713	2.40	0.99180	2.90	0.99813
0.41	0.6591	0.91	0.8186	1.41	0.9207	1.91	0.9719	2.41	0.99202	2.91	0.99819
0.42	0.6628	0.92	0.8212	1.42	0.9222	1.92	0.9726	2.42	0.99224	2.92	0.99825
0.43	0.6664	0.93	0.8238	1.43	0.9236	1.93	0.9732	2.43	0.99245	2.93	0.99831
0.44	0.6700	0.94	0.8264	1.44	0.9251	1.94	0.9738	2.44	0.99266	2.94	0.99836
0.45	0.6736	0.95	0.8289	1.45	0.9265	1.95	0.9744	2.45	0.99286	2.95	0.99841
0.46	0.6772	0.96	0.8315	1.46	0.9279	1.96	0.9750	2.46	0.99305	2.96	0.99846
0.47	0.6808	0.97	0.8340	1.47	0.9292	1.97	0.9756	2.47	0.99324	2.97	0.99851
0.48	0.6844	0.98	0.8365	1.48	0.9306	1.98	0.9761	2.48	0.99343	2.98	0.99856
0.49	0.6879	0.99	0.8389	1.49	0.9319	1.99	0.9767	2.49	0.99361	2.99	0.99861

Table 4.2 The standard normal CDF  $\Phi(y)$ .

$z$	$Q(z)$	$z$	$Q(z)$	$z$	$Q(z)$	$z$	$Q(z)$	$z$	$Q(z)$
3.00	$1.35 \cdot 10^{-3}$	3.40	$3.37 \cdot 10^{-4}$	3.80	$7.23 \cdot 10^{-5}$	4.20	$1.33 \cdot 10^{-5}$	4.60	$2.11 \cdot 10^{-6}$
3.01	$1.31 \cdot 10^{-3}$	3.41	$3.25 \cdot 10^{-4}$	3.81	$6.95 \cdot 10^{-5}$	4.21	$1.28 \cdot 10^{-5}$	4.61	$2.01 \cdot 10^{-6}$
3.02	$1.26 \cdot 10^{-3}$	3.42	$3.13 \cdot 10^{-4}$	3.82	$6.67 \cdot 10^{-5}$	4.22	$1.22 \cdot 10^{-5}$	4.62	$1.92 \cdot 10^{-6}$
3.03	$1.22 \cdot 10^{-3}$	3.43	$3.02 \cdot 10^{-4}$	3.83	$6.41 \cdot 10^{-5}$	4.23	$1.17 \cdot 10^{-5}$	4.63	$1.83 \cdot 10^{-6}$
3.04	$1.18 \cdot 10^{-3}$	3.44	$2.91 \cdot 10^{-4}$	3.84	$6.15 \cdot 10^{-5}$	4.24	$1.12 \cdot 10^{-5}$	4.64	$1.74 \cdot 10^{-6}$
3.05	$1.14 \cdot 10^{-3}$	3.45	$2.80 \cdot 10^{-4}$	3.85	$5.91 \cdot 10^{-5}$	4.25	$1.07 \cdot 10^{-5}$	4.65	$1.66 \cdot 10^{-6}$
3.06	$1.11 \cdot 10^{-3}$	3.46	$2.70 \cdot 10^{-4}$	3.86	$5.67 \cdot 10^{-5}$	4.26	$1.02 \cdot 10^{-5}$	4.66	$1.58 \cdot 10^{-6}$
3.07	$1.07 \cdot 10^{-3}$	3.47	$2.60 \cdot 10^{-4}$	3.87	$5.44 \cdot 10^{-5}$	4.27	$9.77 \cdot 10^{-6}$	4.67	$1.51 \cdot 10^{-6}$
3.08	$1.04 \cdot 10^{-3}$	3.48	$2.51 \cdot 10^{-4}$	3.88	$5.22 \cdot 10^{-5}$	4.28	$9.34 \cdot 10^{-6}$	4.68	$1.43 \cdot 10^{-6}$
3.09	$1.00 \cdot 10^{-3}$	3.49	$2.42 \cdot 10^{-4}$	3.89	$5.01 \cdot 10^{-5}$	4.29	$8.93 \cdot 10^{-6}$	4.69	$1.37 \cdot 10^{-6}$
3.10	$9.68 \cdot 10^{-4}$	3.50	$2.33 \cdot 10^{-4}$	3.90	$4.81 \cdot 10^{-5}$	4.30	$8.54 \cdot 10^{-6}$	4.70	$1.30 \cdot 10^{-6}$
3.11	$9.35 \cdot 10^{-4}$	3.51	$2.24 \cdot 10^{-4}$	3.91	$4.61 \cdot 10^{-5}$	4.31	$8.16 \cdot 10^{-6}$	4.71	$1.24 \cdot 10^{-6}$
3.12	$9.04 \cdot 10^{-4}$	3.52	$2.16 \cdot 10^{-4}$	3.92	$4.43 \cdot 10^{-5}$	4.32	$7.80 \cdot 10^{-6}$	4.72	$1.18 \cdot 10^{-6}$
3.13	$8.74 \cdot 10^{-4}$	3.53	$2.08 \cdot 10^{-4}$	3.93	$4.25 \cdot 10^{-5}$	4.33	$7.46 \cdot 10^{-6}$	4.73	$1.12 \cdot 10^{-6}$
3.14	$8.45 \cdot 10^{-4}$	3.54	$2.00 \cdot 10^{-4}$	3.94	$4.07 \cdot 10^{-5}$	4.34	$7.12 \cdot 10^{-6}$	4.74	$1.07 \cdot 10^{-6}$
3.15	$8.16 \cdot 10^{-4}$	3.55	$1.93 \cdot 10^{-4}$	3.95	$3.91 \cdot 10^{-5}$	4.35	$6.81 \cdot 10^{-6}$	4.75	$1.02 \cdot 10^{-6}$
3.16	$7.89 \cdot 10^{-4}$	3.56	$1.85 \cdot 10^{-4}$	3.96	$3.75 \cdot 10^{-5}$	4.36	$6.50 \cdot 10^{-6}$	4.76	$9.68 \cdot 10^{-7}$
3.17	$7.62 \cdot 10^{-4}$	3.57	$1.78 \cdot 10^{-4}$	3.97	$3.59 \cdot 10^{-5}$	4.37	$6.21 \cdot 10^{-6}$	4.77	$9.21 \cdot 10^{-7}$
3.18	$7.36 \cdot 10^{-4}$	3.58	$1.72 \cdot 10^{-4}$	3.98	$3.45 \cdot 10^{-5}$	4.38	$5.93 \cdot 10^{-6}$	4.78	$8.76 \cdot 10^{-7}$
3.19	$7.11 \cdot 10^{-4}$	3.59	$1.65 \cdot 10^{-4}$	3.99	$3.30 \cdot 10^{-5}$	4.39	$5.67 \cdot 10^{-6}$	4.79	$8.34 \cdot 10^{-7}$
3.20	$6.87 \cdot 10^{-4}$	3.60	$1.59 \cdot 10^{-4}$	4.00	$3.17 \cdot 10^{-5}$	4.40	$5.41 \cdot 10^{-6}$	4.80	$7.93 \cdot 10^{-7}$
3.21	$6.64 \cdot 10^{-4}$	3.61	$1.53 \cdot 10^{-4}$	4.01	$3.04 \cdot 10^{-5}$	4.41	$5.17 \cdot 10^{-6}$	4.81	$7.55 \cdot 10^{-7}$
3.22	$6.41 \cdot 10^{-4}$	3.62	$1.47 \cdot 10^{-4}$	4.02	$2.91 \cdot 10^{-5}$	4.42	$4.94 \cdot 10^{-6}$	4.82	$7.18 \cdot 10^{-7}$
3.23	$6.19 \cdot 10^{-4}$	3.63	$1.42 \cdot 10^{-4}$	4.03	$2.79 \cdot 10^{-5}$	4.43	$4.71 \cdot 10^{-6}$	4.83	$6.83 \cdot 10^{-7}$
3.24	$5.98 \cdot 10^{-4}$	3.64	$1.36 \cdot 10^{-4}$	4.04	$2.67 \cdot 10^{-5}$	4.44	$4.50 \cdot 10^{-6}$	4.84	$6.49 \cdot 10^{-7}$
3.25	$5.77 \cdot 10^{-4}$	3.65	$1.31 \cdot 10^{-4}$	4.05	$2.56 \cdot 10^{-5}$	4.45	$4.29 \cdot 10^{-6}$	4.85	$6.17 \cdot 10^{-7}$
3.26	$5.57 \cdot 10^{-4}$	3.66	$1.26 \cdot 10^{-4}$	4.06	$2.45 \cdot 10^{-5}$	4.46	$4.10 \cdot 10^{-6}$	4.86	$5.87 \cdot 10^{-7}$
3.27	$5.38 \cdot 10^{-4}$	3.67	$1.21 \cdot 10^{-4}$	4.07	$2.35 \cdot 10^{-5}$	4.47	$3.91 \cdot 10^{-6}$	4.87	$5.58 \cdot 10^{-7}$
3.28	$5.19 \cdot 10^{-4}$	3.68	$1.17 \cdot 10^{-4}$	4.08	$2.25 \cdot 10^{-5}$	4.48	$3.73 \cdot 10^{-6}$	4.88	$5.30 \cdot 10^{-7}$
3.29	$5.01 \cdot 10^{-4}$	3.69	$1.12 \cdot 10^{-4}$	4.09	$2.16 \cdot 10^{-5}$	4.49	$3.56 \cdot 10^{-6}$	4.89	$5.04 \cdot 10^{-7}$
3.30	$4.83 \cdot 10^{-4}$	3.70	$1.08 \cdot 10^{-4}$	4.10	$2.07 \cdot 10^{-5}$	4.50	$3.40 \cdot 10^{-6}$	4.90	$4.79 \cdot 10^{-7}$
3.31	$4.66 \cdot 10^{-4}$	3.71	$1.04 \cdot 10^{-4}$	4.11	$1.98 \cdot 10^{-5}$	4.51	$3.24 \cdot 10^{-6}$	4.91	$4.55 \cdot 10^{-7}$
3.32	$4.50 \cdot 10^{-4}$	3.72	$9.96 \cdot 10^{-5}$	4.12	$1.89 \cdot 10^{-5}$	4.52	$3.09 \cdot 10^{-6}$	4.92	$4.33 \cdot 10^{-7}$
3.33	$4.34 \cdot 10^{-4}$	3.73	$9.57 \cdot 10^{-5}$	4.13	$1.81 \cdot 10^{-5}$	4.53	$2.95 \cdot 10^{-6}$	4.93	$4.11 \cdot 10^{-7}$
3.34	$4.19 \cdot 10^{-4}$	3.74	$9.20 \cdot 10^{-5}$	4.14	$1.74 \cdot 10^{-5}$	4.54	$2.81 \cdot 10^{-6}$	4.94	$3.91 \cdot 10^{-7}$
3.35	$4.04 \cdot 10^{-4}$	3.75	$8.84 \cdot 10^{-5}$	4.15	$1.66 \cdot 10^{-5}$	4.55	$2.68 \cdot 10^{-6}$	4.95	$3.71 \cdot 10^{-7}$
3.36	$3.90 \cdot 10^{-4}$	3.76	$8.50 \cdot 10^{-5}$	4.16	$1.59 \cdot 10^{-5}$	4.56	$2.56 \cdot 10^{-6}$	4.96	$3.52 \cdot 10^{-7}$
3.37	$3.76 \cdot 10^{-4}$	3.77	$8.16 \cdot 10^{-5}$	4.17	$1.52 \cdot 10^{-5}$	4.57	$2.44 \cdot 10^{-6}$	4.97	$3.35 \cdot 10^{-7}$
3.38	$3.62 \cdot 10^{-4}$	3.78	$7.84 \cdot 10^{-5}$	4.18	$1.46 \cdot 10^{-5}$	4.58	$2.32 \cdot 10^{-6}$	4.98	$3.18 \cdot 10^{-7}$
3.39	$3.49 \cdot 10^{-4}$	3.79	$7.53 \cdot 10^{-5}$	4.19	$1.39 \cdot 10^{-5}$	4.59	$2.22 \cdot 10^{-6}$	4.99	$3.02 \cdot 10^{-7}$

Table 4.3 The standard normal complementary CDF  $Q(z)$ .